Vibrational resonance and the detection of aperiodic binary signals

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We present the experimental and numerical study of a method for detecting aperiodic binary signals in a bistable, vertical cavity surface emitting laser (VCSEL). The method uses the phenomenon of vibrational resonance, in presence of a fixed level of noise. We show that the addition of a periodic signal with a period much shorter than the bit duration of the aperiodic input signal allows one to significantly increase the cross-correlation coefficient between the input and the output, as well as to substantially decrease the bit error rate. The experimental observation of a time lag between the input and the output of the VCSEL due the high-frequency modulation is reported. The effect of an asymmetry of the bistable quasipotential on the detection is also analyzed. The numerical results of simulations in a simple model are in qualitative agreement with the experiment.

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I. INTRODUCTION

The dynamics of an overdamped bistable oscillator driven simultaneously by two periodic signals with strongly different frequencies recently becomes a subject of broad interest both theoretically and experimentally [1–9]. In such conditions, the response of the system to a weak low-frequency excitation passes through a maximum depending on the amplitude or frequency of a high-frequency (HF) signal. The phenomenon has been named "vibrational resonance" (VR) [1]. The mechanism underlying VR is the parametric amplification of signals and fluctuations near a bifurcation, corresponding to the transition from bistability to monostability controlled by the HF signal [2,4,6,7]. Experimentally the phenomenon of VR has been revealed in analog electric circuits [2] and in the polarization dynamics of bistable semiconductor surface-emitting vertical-cavity laser [3]. In such a situation, it was shown that the gain factor and the signal-to-noise ratio (SNR) are higher than using the phenomenon of stochastic resonance (SR) [6,7]; moreover, it was experimentally and theoretically demonstrated for subthreshold square-wave, periodic signals that an output SNR greater than the input one can be obtained in a wide range of noise intensity [7].

Recently, similar results for a subthreshold, sinusoidal signals were obtained theoretically [9], while other theoretical investigations have revealed the occurrence of VR in a system of two coupled overdamped oscillators [10]. The control of synchronization at the low frequency by a HF excitation were also demonstrated in a noisy Schmidt trigger [11]. Among other recent results, the effect of the HF excitation on a mobility of a Brownian particle in the ratchet [12] and on the neuron activity in the FitzHugh-Nagumo excitable model [13] were theoretically studied. It should be noted that all these studies on VR were performed in the case of periodic input signals; therefore, an extension of such an approach for the improvement of detection and recovery of different types of signals represents a significant scientific and practical goal.

The problem of improving the detection of noisy aperiodic signals in bistable systems is strictly related to the phenomenon of stochastic resonance and it has been shown in a number of theoretical and experimental investigations [14–22]. Such a topic was named "aperiodic stochastic resonance" (ASR) [14]. While in the case of SR the usual measure of the detection quality is the signal-to-noise ratio, in ASR the cross-correlation coefficient between input and output signals [14–16], the mutual information [17,18], the information capacity [19] of the channel, and the bit error rate (BER) [20–22] are used as a rule.

In this paper we present the experimental and numerical investigation of the effect of VR when detecting an aperiodic binary signals using a VCSEL. The laser operates in the regime of the polarization bistability, in both the nearly symmetric and asymmetric configurations of the double-well quasipotential. A measure of the quality of the detection is carried out using the cross-correlation coefficient and the BER.

II. EXPERIMENTAL SETUP

The experimental setup was almost the same used for previous investigations of VR in VCSEL [3]. In the experiments described below we used a VCSEL lasing at 840 nm. The laser was thermally stabilized with an accuracy of a few mK. The injection current was chosen so that the laser operates in the regime of polarization bistability where only very rare switchings induced by internal noise are observed. We studied the response in the laser intensity after polarization selection when the mixture of the aperiodic binary signal and the periodic signal were applied to the injection current. In this configuration, we injected a pseudorandom binary signal with 10 kB/s bit rate with different subthreshold amplitudes A_L and a square-wave HF control signal with frequency $f_H = 200$ kHz and amplitude A_H, which represents one of the main control parameters of the experiment. The aperiodic

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signal used $R(t)$ is a sequence of constant values on consecutive intervals with the same duration $T_b = 100\ \mu s$. The value of $R(t)$ in each interval is chosen to be +1 or −1 according to a random process with probability 0.5. We call such a sequence a (pseudorandom) bit stream. In the experiment here reported, at variance with the previous setup [3], we did not add noise to the injection current and only the intrinsic spontaneous emission noise of VCSEL is acting. All measurements were performed with such fixed level of noise.

The laser responses were detected with a fast photodetector and recorded by a digital, 50 MHz-bandwidth USB oscilloscope coupled with a computer to store and process the data. The values of the analyzed statistical indicators were obtained by averaging over 10 to 500 measurements, each counting 130000 points sampled with a period of 1 μs.

III. MODEL

Most of the features of the dynamics of a VCSEL working in a bistable regime can be well described by a simple model of an overdamped oscillator with a double-well quasipotential (see, e.g., Refs. [23–25]). We performed a numerical simulation in the framework of the equation

$$dx/dt = 4(x - x^3) + \Delta + A_L R(t) + A_H \operatorname{sgn}(\sin \Omega_H t) + \zeta(t), \tag{1}$$

where Δ is a level of the asymmetry, $R(t)$ represents the aperiodic binary signal (binary string) used in the experiments, $\operatorname{sgn}(x)$ is a signum function $\operatorname{sgn}(x) = -1$, for $x < 0$, and $\operatorname{sgn}(x) = 1$, for $x > 0$, $\zeta(t)$ is a white, Gaussian noise with $\langle \zeta(t)\zeta(t')\rangle = 2D\delta(t-t')$ and zero mean $\langle \zeta(t)\rangle = 0$. A forward Euler algorithm with a fixed step of $0.001T_H$ was used ($T_H = 2\pi/\Omega_H$).

For a quantitative characterization of aperiodic vibrational resonance, the normalized cross correlation coefficient between the input and output signals were evaluated from time series for each amplitude of the HF signal

$$C_{IO}(\tau) = \frac{\langle R(t)I(t+\tau)\rangle - \langle R(t)\rangle\langle I(t+\tau)\rangle}{\{[\langle R^2(t)\rangle - \langle R(t)\rangle^2][\langle I^2(t)\rangle - \langle I(t)\rangle^2]\}^{1/2}}, \tag{2}$$

where $R(t)$ represents an aperiodic binary signal, $I(t)$ is a response of a bistable system. $\langle \cdots \rangle$ represents the time average. The indicator (2) was used in order to evaluate the maximal value of $C_{IO}(\tau)$ and to estimate the time lag between the signals in the processing of both experimental and numerical time series.

IV. RESPONSE OF VCSEL'S TO PERIODIC MODULATION

First of all, we have investigated the switching thresholds between polarization states in the bistable regime for rectangular periodic signals with different frequencies. In Fig. 1 we show the response of the laser, obtained from the Fourier spectrum of time series, as a function of the amplitude of applied periodic modulation for the two frequencies $f_m = 10$ kHz (curve 1) and $f_m = 200$ kHz (curve 2). The frequency $f_m = 10$ kHz corresponds to the highest base frequency in the aperiodic binary signal. From these curves we can estimate the switching thresholds $\mu_L \approx 3.1$ mV (at $f_m = 10$ kHz) and $\mu_H \approx 7.5$ mV (at $f_m = 200$ kHz), the uncertainty due to the unavoidable presence of noise in a real system. The value μ_L will be used for the normalization of the amplitude of the binary signal. We have also experimentally studied the influence of the asymmetry of the quasipotential on the switching threshold for the same modulation frequencies. We found that there exists a minimal threshold for both frequencies, that corresponds to nearly symmetric configuration of the bistable quasipotential. Therefore, in order to ensure a nearly symmetrical configuration in experimental investigations, we tuned the injection current to this value.

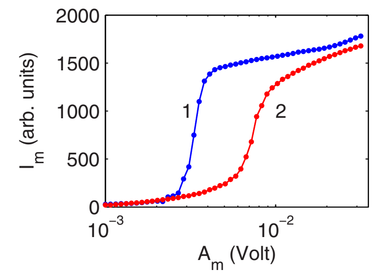

FIG. 1. (Color online) Experiment. Polarization-resolved response of VCSEL I_m to a square-wave periodic modulation as a function of the amplitude V_m, shown for two different modulation frequencies $f_m = 10$ kHz (1) and 200 kHz (2).

V. APERIODIC MODULATION

In this section, we analyze in detail the effect of the addition of HF modulation in a VR regime, when using a binary aperiodic signal for the case of the symmetric bistable potential. In Fig. 2(a) we show (a part of) the aperiodic binary signal used in the experimental (and numerical) inves-

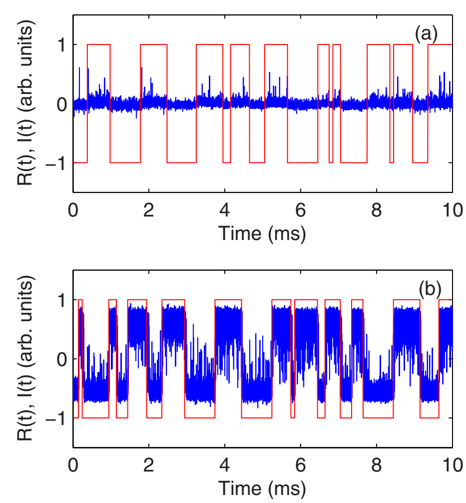

FIG. 2. (Color online) Experiment. Temporal behavior of polarization-resolved intensity of VCSEL [thick (blue) line]: (a) in the absence and (b) in the presence of the additional HF modulation with the optimal amplitude. The thin (red) line is an input aperiodic binary signal with subthreshold amplitude $\epsilon \approx 0.16$.

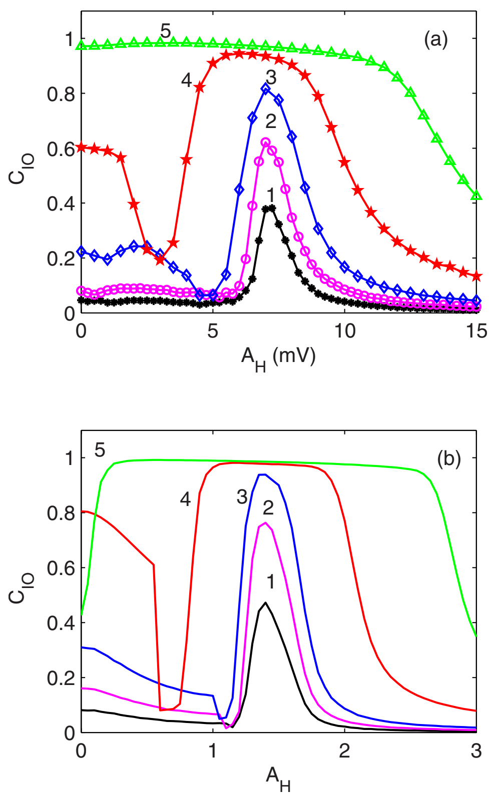

FIG. 3. (Color online) The cross-correlation coefficient C_{IO} as a function of the amplitude of the HF modulation A_H, shown for different values of the amplitude of binary signal (normalized to the switching threshold). (a) Experiment: $\epsilon \approx 0.02$ (1), 0.04 (2), 0.08 (3), 0.25 (4), 0.67 (5). (b) Numerical simulation: $\Delta = 0$. $\epsilon \approx 0.02$ (1), 0.04 (2), 0.08 (3), 0.32 (4), 0.8 (5).

tigations and the response of VCSEL in the absence of the additional HF modulation. In this case, a binary signal amplitude $A_L = 0.5$ mV (or $\epsilon = A_L/\mu_L \approx 0.16$ when normalized to the switching threshold) was used. The addition of a HF signal with the optimal amplitude to the injection current of the laser leads to (nearly) perfect recovering the aperiodic binary signal with a significant increase of its amplitude [Fig. 2(b)].

For a quantitative estimation of a quality of the detection, we performed a set of measurements of the cross-correlation coefficient as a function of the amplitude of the HF signal, for different amplitudes of the aperiodic binary signal. The experimental results are shown in Fig. 3(a), where we represent the maximum of the cross-correlation $C_{IO} = \max\{C_{IO}(\tau)\}$ as a function of the HF amplitude. For comparison purposes, in Fig. 3(b) we also present the results of the numerical simulation. In the simulation reported in Eq. (1) we introduced a small level of noise intensity ($D = 0.082$), in order to obtain a qualitative agreement between experimental and numerical results.

First of all, one can note that C_{IO} passes through the maximum as the amplitude of the HF signal increases. Such a behavior is a manifestation of VR for the aperiodic binary signal and it is observed for all dependencies presented in the figure. It should be noted the phenomenon is clearly observed even for very weak input binary signal [$\epsilon \approx 0.02$, curve 1 in Fig. 3(a)]. One can note also the broadening of curves and the shift of the optimal value of A_H as the amplitude of the binary signal increases.

These measurements are in a good agreement with the numerical results shown in Fig. 3(b). The location of the maximum of the curve is in the vicinity of the switching threshold μ_H, but slightly shifted to the lower values of A_H due to the presence of internal noise in the laser. Such a shift strongly depends on the noise intensity and can be large enough to be significant, as we have found investigating the effect of noise on the $C_{IO}^{\max} = \max\{C_{IO}\}$ in the simulations (see below). The numerical analysis shows that for the given amplitude of the binary signal, the noise significantly diminishes $C_{IO}^{\max}$, worsening the quality of the signal. In this case, a diminution of $C_{IO}^{\max}$ obeys a scaling law similar to that found in Ref. [6] in the case of noise-induced gain degradation for periodic signals in VR: $C_{IO}^{\max} \sim D^{-\gamma}$, where γ lies between 0 and 0.5 and tends to 0.5 as A_L goes to 0.

In Figs. 4(a) and 4(b) we report the experimental and numerical dependencies of $C_{IO}^{\max}$ (curve 1 on both figures), respectively, as a function of the normalized amplitude of the aperiodic binary signal. For comparison, the same indicator in the absence of HF driving is shown (C_{in}, curve 2 in both figures). It is evidently a great improvement of the cross-correlation coefficient. We remark also on the good qualitative agreement between experimental and numerical results.

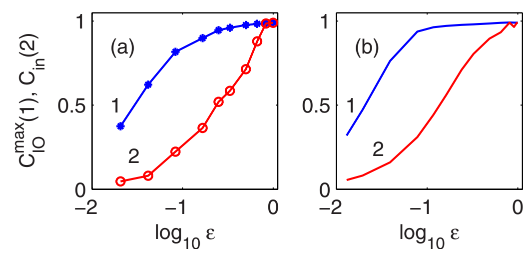

FIG. 4. (Color online) $C_{IO}^{\max}$ in the presence (1) and in the absence (2) of the HF modulation as a function of the amplitude of aperiodic binary signal ϵ. (a) Experiment. (b) Simulation ($\Delta = 0$).

VI. TIME LAG BETWEEN THE INPUT AND THE OUTPUT

An important feature of VR is the existence of a time lag between the input and output signals, as shown numerically in Ref. [1]. Until now, such an effect was not observed experimentally. In Fig. 5(a) we present the evidence of such time lag between the input aperiodic binary signal and the output signal from the laser. We have used a normalized time lag δT defined as $\delta T = \tau_m / T_b$, where τ_m is a value of τ corresponding to the maximum of $C_{IO}(\tau)$ for the given HF amplitude.

It should be noted that such a lag between input and output signals was shown in stochastic resonance (phase shift) [26]. Linear response theory and analog experiments demonstrated that such shift passes through a maximum depending on the applied noise intensity. In the case of vibrational resonance, the nonmonotonic behavior is observed versus the amplitude of the HF modulation and it is characterized by

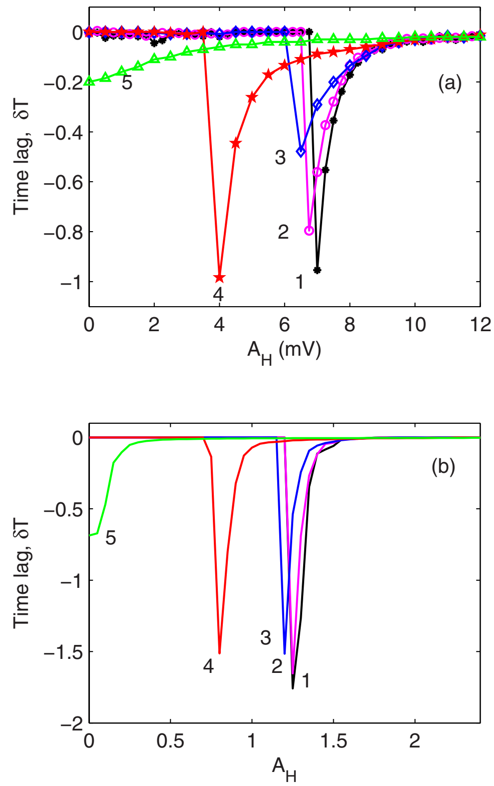

FIG. 5. (Color online) (Normalized) time lag δT between the input aperiodic signal and the response of the system as a function of the amplitude of the HF modulation A_H, for different values of the amplitude of the aperiodic binary signal. (a) Experiment: $\epsilon \approx 0.02$ (1), 0.04 (2), 0.08 (3), 0.25 (4), 0.67 (5). (b) Numerical simulation: $\Delta=0$. $\epsilon \approx 0.02$ (1), 0.04 (2), 0.08 (3), 0.32 (4), 0.8 (5).

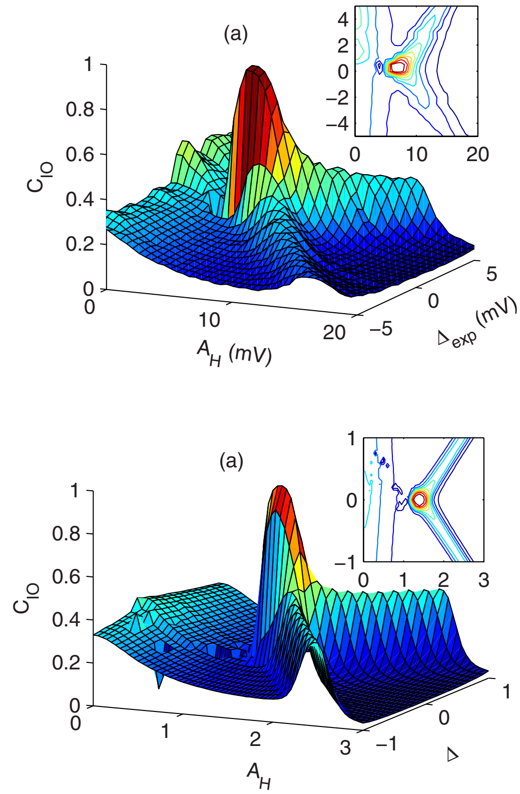

FIG. 6. (Color online) The coefficient of cross-correlation C_{IO} for aperiodic binary signal in the presence of noise as a function of the amplitude of the HF modulation A_H and the level of asymmetry Δ (Δ_{exp}) of a bistable quasipotential. (a) Experiment ($\epsilon \approx 0.16$); (b) Numerical simulation ($\epsilon = 0.16$). Inset in both figures: the contour plot for $C_{IO}^{\max}$ as a function A_H and Δ (Δ_{exp}).

the appearance of the discontinuity in the initial part of the dependence (see Fig. 5).

VII. EFFECT OF ASYMMETRY

Most of studies on VR were concentrated on symmetrical bistable potentials, except Ref. [8], where the effect of asymmetry was investigated in detail both experimentally and theoretically. This effect if of great importance from the standpoint of possible applications of VR in the communication system, since the robustness of the phenomenon must be verified in a real-world situation where the asymmetries of the bistable potential are always present and should be taken into account.

Here, we proceed in this analysis in the case of aperiodic binary signal. We report in Figs. 6 and 7 the comparison between the measurements and the results of numerical simulations.

In Fig. 6(a) we show the three-dimensional plots of C_{IO} as a function of the HF amplitude A_H and the level of asymmetry Δ, for the experimental conditions defined as $\Delta_{\text{exp}}=V-V_0$, where V is a current value of the applied voltage and V_0 is the applied voltage corresponding to the symmetrical quasipotential. In these experiments, we changed the injection current (with a step of 0.5 mV) and the level of asymmetry of the quasipotential [23]. We can notice how the maximum of C_{IO} for a weak input signal can be obtained only in narrow ranges of both A_H and Δ_{exp}. These experimental results are well supported by the numerical results shown in Fig. 6(b). We performed a series of similar measurements with

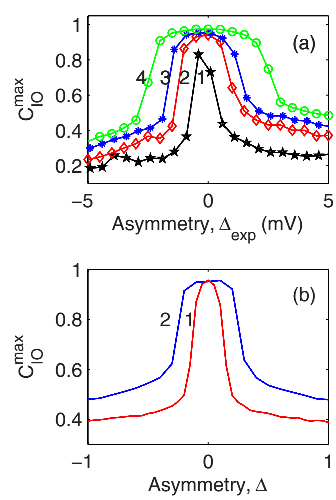

FIG. 7. (Color online) The coefficients of cross-correlation $C_{IO}^{\max}$ as a function of the level asymmetry Δ_{exp} (Δ). (a) Experiment: $\epsilon = 0.1$ (1), 0.25 (2), 0.33 (3), 0.5 (4). (b) Numerical simulation: $\epsilon = 0.1$ (1), 0.2 (2).

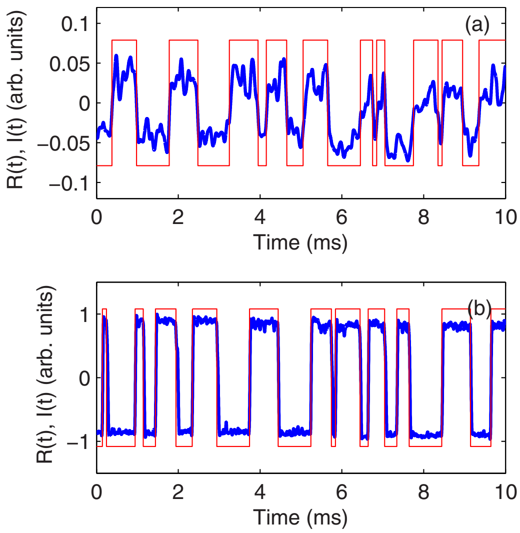

FIG. 8. (Color online) Experiment. A filtered polarization-resolved laser response [thick (blue) line]: (a) in the absence and (b) in the presence of the additional HF modulation with the optimal amplitude. Thin (red) line is an input aperiodic binary signal with subthreshold amplitude $\epsilon \approx 0.16$.

different amplitudes of the aperiodic signal, summarized in Fig. 7(a). There, the behavior of $C_{IO}^{\max}$ is shown as a function of the level of asymmetry for different values of the amplitude of the aperiodic binary signal. As a result, the asymmetry leads to a strong diminution of $C_{IO}^{\max}$ when $A_L < \Delta_{\exp}$.

Such a rather strong decrease of the cross-correlation coefficient with increasing the level of the asymmetry can be explained by the different temporal behavior of a bistable system when the quasipotential changes from symmetrical to strongly asymmetrical. In the former case, with an optimal amplitude of the HF modulation, the bistable system output (to periodic or aperiodic modulations) almost reproduces the shape of the input signal with some minor variations of the period or of the bit duration due to the presence noise. In the latter case, the response is filled out by HF modulation pulses of one sign only. As a result, both the gain factor for the periodic signal and the cross-correlation coefficient for the aperiodic signals decrease (pictures of the temporal behavior are shown in Ref. [3]).

VIII. IMPROVEMENT OF CORRELATION BY LOW-PASS FILTERING

All the experimental results presented above were obtained without additional processing of time series. The response of the bistable VCSEL can be roughly separated into three parts, namely, aperiodic, HF components, and noise. Since the time scales of deterministic signals are very different, one can expect that a simple low-pass filtering can further improve the correlation coefficient. For this purpose, we used a software-implemented acausal low-pass filter with the transfer function $H(\omega)=1/(1+\alpha\omega^4)$, where the parameter α was chosen in order to maximize the input-output cross-correlation. In Fig. 8(a) ($\alpha=1.2\times10^{-3}$) and Fig. 8(b) ($\alpha=0.43\times10^{-3}$) we show the optimally filtered responses of VCSEL in the absence (presence) of the HF modulation with the optimal amplitude. Comparing the two signals, one can note a very good recovery of the input signal with a substantial increase of its amplitude.

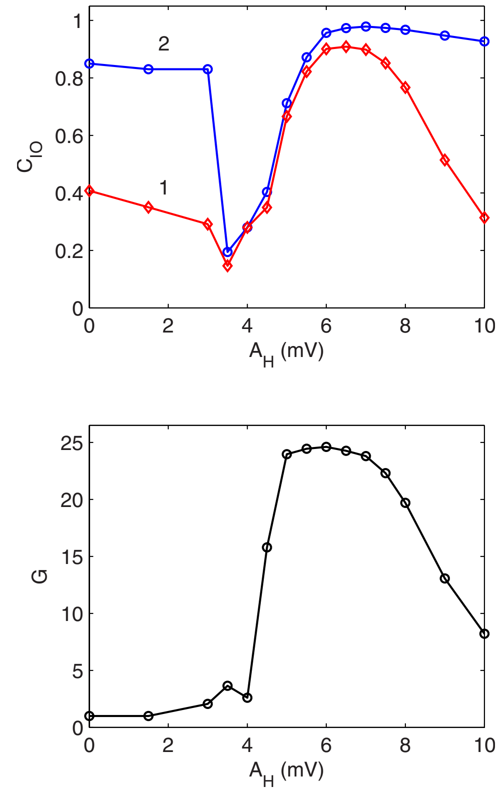

FIG. 9. (Color online) Experiment. (a) C_{IO} as a function of A_H: 1: raw data, 2: filtered data. (b) The amplification of the amplitude G as a function of A_H. ($\epsilon \approx 0.16$).

After the filtering process, one can characterize the gain of the amplitude induced by the HF modulation. We introduce the quantity g_c defined as $g_c=\langle \tilde{I}_+\rangle - \langle \tilde{I}_-\rangle$, where $\tilde{I}$ is the filtered response (time series) after subtracting the mean value $\langle I(t)\rangle$, $\tilde{I}_+$ and $\tilde{I}_-$ are all positive and all negative values of $\tilde{I}$, respectively. The quantity g_c represents a signal amplitude averaged over the whole time series. The normalized value $G=g_c/g_0$ defines the amplification of the amplitude with respect to the absence of the HF signal (g_0 is the amplitude of the laser response in the absence of the HF modulation). In Fig. 9(a) we show the improved C_{IO} (open circles) and C_{IO} obtained without filtering (diamonds). Low-pass filtering increases C_{IO} by about 7% in the range of A_H corresponding to VR. After filtering, $C_{IO}^{\max}$ reaches the value of ≈ 0.98 whereas with raw data (without filtering) $C_{IO}^{\max} \approx 0.91$ at the optimal amplitude of the HF modulation. At the same time, the amplitude of the laser response increases by about 25 times [Fig. 9(b)] with respect to the initial value (in the absence of the HF modulation).

IX. BIT ERROR RATE

We present in Fig. 10 the experimental results showing the measurements of bit error rate in the system considered

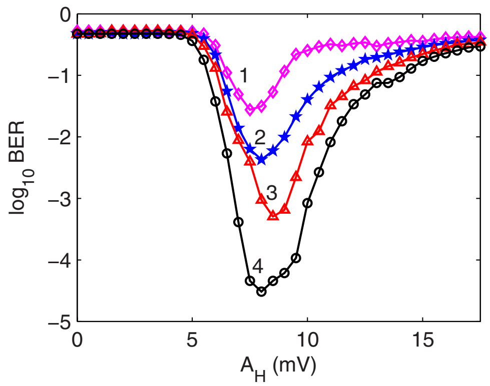

FIG. 10. (Color online) Experiment. Bit error rate (BER) as a function of the amplitude A_H for different values of the normalized amplitude of the aperiodic binary signal [$\epsilon \approx 0.1(1)$, 0,15 (2), 0.2 (3), 0.25 (4)].

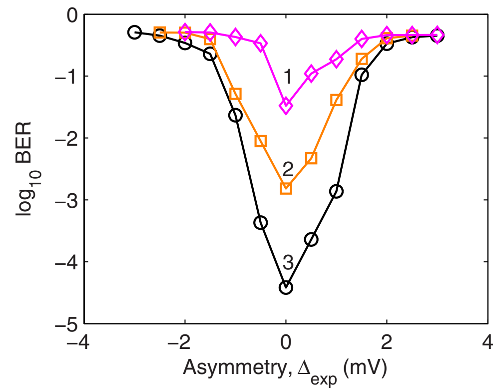

FIG. 11. (Color online) Experiment. Bit error rate (BER) as a function of the level asymmetry Δ_{exp}, for different values of the normalized amplitude of the aperiodic binary signal [$\epsilon \approx 0.1$ (1), 0.18 (2), 0.25 (3)].

as a communication device. The BER is the most commonly used measure for an evaluation of the quality of the data transmission through communication channels and it is defined as the percentage of wrong received bits with respect to the total number of transmitted bits. Depending on the applications, a BER from 10^{-2} to less than 10^{-8} is required. In processing our experimental data, the BER was evaluated by averaging the output signal over the input bit length in a similar fashion as in Ref. [22]. One can note a significant decrease of BER at the optimal value of the HF amplitude. For instance, for $\epsilon = 0.25$ (a quarter of the switching threshold) the minimal BER is about 3×10^{-5}, a value which cannot be reached in aperiodic stochastic resonance for this amplitude of the aperiodic binary signal. We also studied the effect of asymmetry of quasipotential on the BER and found the same regularities as for a cross-correlation coefficient presented above, as shown in the following. The BER is plotted in Fig. 11 as a function of the level of asymmetry for three different values of the amplitude of the aperiodic binary signal. One can note that an increase of the asymmetry leads to a strong increase of the BER (thus, to a deteriorated quality of the fidelity of the transmission) and its effect shows up again when the amplitude of the aperiodic signal $A_L < \Delta$.

X. CONCLUSIONS

We have shown that vibrational resonance is an effective method to improve the detection and recovery of weak subthreshold aperiodic binary signals in stochastic bistable systems. We have demonstrated that such an approach allows us to greatly increase the coefficient of cross-correlation between the input and the output signals and reduce the bit error rate. With respect to recently proposed methods of tuning system parameters or parameter-induced aperiodic stochastic resonance [27–29], it does not require one to change the internal parameters of the bistable systems. Since vibrational resonance is based on the existence of very different time scales of the input and the control signal, the HF component can be efficiently removed by an additional low-pass filter with a further improvement in detection. We believe that the results we reported will be useful in applications relating to the detection and the recovering of weak aperiodic binary signals, such as in the field of pattern recognition, where cross-correlation is used as a measure of similarity between input and reference patterns and in the transmission of digital and analog data in noisy communication channels.

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[1] P. S. Landa and P. V. E. McClintock, J. Phys. A **33**, L433 (2000).
[2] J. P. Baltanas, L. Lopez, I. I. Blechman, P. S. Landa, A. Zaikin, J. Kurths, and M. A. F. Sanjuan, Phys. Rev. E **67**, 066119 (2003).
[3] V. N. Chizhevsky, E. Smeu, and G. Giacomelli, Phys. Rev. Lett. **91**, 220602 (2003).
[4] I. I. Blekhman and P. S. Landa, Int. J. Non-linear Mech. **39**, 421 (2004).
[5] J. Casado-Pascual and J. P. Baltanas, Phys. Rev. E **69**, 046108 (2004).
[6] V. N. Chizhevsky and G. Giacomelli, Phys. Rev. E **70**, 062101 (2004).
[7] V. N. Chizhevsky and G. Giacomelli, Phys. Rev. A **71**, 011801(R) (2005).
[8] V. N. Chizhevsky and G. Giacomelli, Phys. Rev. E **73**, 022103 (2006).
[9] J. Casado-Pascual, D. Cubero, and J. P. Baltanas, Europhys. Lett. **77**, 50004 (2007).
[10] V. M. Gandhimathi, S. Rajasekar, and J. Kurths, Phys. Lett. A **360**, 279 (2006).
[11] I. A. Khovanov and P. V. E. McClintock, Phys. Rev. E **76**,

031122 (2007).

[12] M. Borromeo and F. Marchesoni, Europhys. Lett. **72**, 362 (2005).

[13] D. Cubero, J. P. Baltanas, and J. Casado-Pascual, Phys. Rev. E **73**, 061102 (2006).

[14] J. J. Collins, C. C. Chow, and T. T. Imhoff, Phys. Rev. E **52**, R3321 (1995).

[15] J. J. Collins, C. C. Chow, A. C. Capela, and T. T. Imhoff, Phys. Rev. E **54**, 5575 (1996).

[16] S. Barbay, G. Giacomelli, and F. Marin, Phys. Rev. Lett. **85**, 4652 (2000).

[17] A. R. Bulsara and A. Zador, Phys. Rev. E **54**, R2185 (1996).

[18] M. Misono, T. Kohmoto, Y. Fukuda, and M. Kunitomo, Phys. Rev. E **58**, 5602 (1998).

[19] F. Chapeau-Blondeau, Phys. Rev. E **55**, 2016 (1997).

[20] Hu Gang, Gong De-chun, Wen Xiao-dong, Yang Chun-yuan, Qing Guang-rong, and Li Rong, Phys. Rev. A **46**, 3250 (1992).

[21] S. Barbay, G. Giacomelli, and F. Marin, Opt. Lett. **25**, 1095 (2000).

[22] S. Barbay, G. Giacomelli, and F. Marin, Phys. Rev. E **63**, 051110 (2001).

[23] G. Giacomelli and F. Marin, Quantum Semiclassic. Opt. **10**, 469 (1998).

[24] M. B. Willemsen, M. U. F. Khalid, M. P. van Exter, and J. P. Woerdman, Phys. Rev. Lett. **82**, 4815 (1999).

[25] T. D. Frank, M. Sondermann, T. Ackemann, and R. Friedrich, Nonlinear Phenom. Complex Syst. (Dordrecht, Neth.) **8**, 193 (2005).

[26] M. I. Dykman, R. Mannella, P. V. E. McClintock, and N. G. Stocks, Phys. Rev. Lett. **68**, 2985 (1992).

[27] B. Xu, F. Duan, and F. Chapeau-Blondeau, Phys. Rev. E **69**, 061110 (2004).

[28] B. Xu, J. Li, and J. Zheng, Physica A **343**, 156 (2004).

[29] L. Jian-Long, Chin. Phys. **16**, 340 (2007).